PHILOSOPHICAL IMPLICATIONS OF EMMY NOETHER'S THEOREM – CONNECTION OF SYMMETRIES AND CONSERVATION LAWS

Dragan Poljak, Mirko Jakić
University of Split, FESB

Scope

- There are physical quantities that remain constant throughout the time evolution of a physical system.
- Such quantities are regarded to be conserved under certain conditions which is governed by corresponding conservation laws.
- Conservation laws are consequence of the symmetry properties of a physical system, i.e. invariance properties of a system.
- These symmetry properties of the system and related conservation laws stem from Emmy Noether's theorem.

Scope

- When a functional i.e. *action*, is extremal, Emmy Noether's theorem yields the conservation law.
- Thus, invariance of the system under a time translation leads to the energy conservation, a space translation invariance corresponds to the conservation of linear momentum, rotation invariance corresponds to the conservation of angular momentum, while gauge invariance yields the conservation of electric charge.

Scope

- From the philosophical point of view, one deals with a path of unification of mathematics and physics, even with connection of the epistemic and ontic aspects, as fundamental laws of nature are governed by the equations having a beautiful inherent symmetry.

- This work deals with philosophical and physical implications of Emmy Noether's theorem.

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.

Eugene Wigner

Introduction

- Problem of connection of matehmatics and real world, i.e. (un)reasonable effectiveness of matehmatics in natural sciences still initiates long and deep philosophical disccussions.
- Modern physics explains the conservation of charge in electromagnetism by using the local symmetry, so called gauge symmetry, which essentially stems from mathematics.

Introduction

- Many researchers and philosophers address mathematical models simply with kind of instrumentalism, which could be always extended and improved, and they actually deny a connection of mathematics with objective reality.
- Bohr claims that models are useful tool to make predictions of the measurement outcomes and he does not connect these models with reality, itself.
- Bohr's attitude is in opposition to Einstein's God who does not play dice.

Introduction

- Physical laws can be described by the action while the symmetry of the law corresponds to the invariance of the action under various transformations.
- According to Noether's theorem, certain action symmetry corresponds to a conserved quantity.
- Namely, there are physical quantities that do not change throughout the time evolution of a physical systems.
- These quantities are then stated to be conserved under certain conditions which is governed by conservation laws.

Introduction

- Conservation laws are consequence of the symmetry properties of a physical system (invariance properties of a system under a group of transformations).
- These symmetry properties of the system and conservation laws are connected with Noether's theorem.
- Noether's 1st theorem deals with global symmetries and conservation laws stating that for every continuous global symmetry there is a conservation law.
- Continuous local symmetries depend on arbitrary functions of the spacetime coordinates.

Introduction

- Important consequences of Noether's 1st theorem are evident in classical mechanics, i.e spatial translation corresponds to linear momentum, spatial rotation to angular momentum and time translation to energy conservation, respectively.
- In electromagnetics, on the other hand an important consequence is the correspondence between gauge symmetry and electric charge conservation.

Introduction

- Mathematically, one obtains the link between the symmetry and the continuity equation for charge conservation provided the equation of motions are satisfied.
- This work deals with a philosophical view associated with relationship of mathematics and physics within a framework given by Noether's theorem.

Least action principle

- The mathematical base for Noether's theorem is calculus of variations.
- While functions represents a mapping from set of real numbers to real numbers, functionals are mappping from set of certain functions to the set of real numbers.
- Many real-world problems arising in science and technology can be solved only by the use of numerical methods which dominantly stem from variational approaches.

Least action principle

- In classical mechanics one may use equations of motion in a Newtonian sense to predict the evolution of the systems provided some initial conditions and boundary conditons at the present moment are known.
- Alternatively one may specify starting and final position, or inital and final time instant and seek for the trajectory satisfying the minimum energy, or minimum time request.

Least action principle

- The solution, i.e. the obtained trajectory, minimizes the difference between the kinetic and potential energy of the system integrated over time.
- This is the well-known the ***least action principle.***
- Note that time integral of the difference between kinetic energy and potential energy is called ***action.***

Least action principle

- Historically, the least action principle comes from Maupertuis and Fermat and can be illustrated on the phenomena of light reflection and refraction, respectively.
- When considering light ray propagation, it appears that among all possible paths between the initial instant t_1 and final instant t_2 light ray follows the path that takes the least time to travel from t_1 to t_2.

Least action principle

- Fig 1. shows the reflection of light ray. The shortest, and the fastest, path is the one having the same angle of incidence and reflection (path ACB). Any other path (A'CB) is necessary longer.

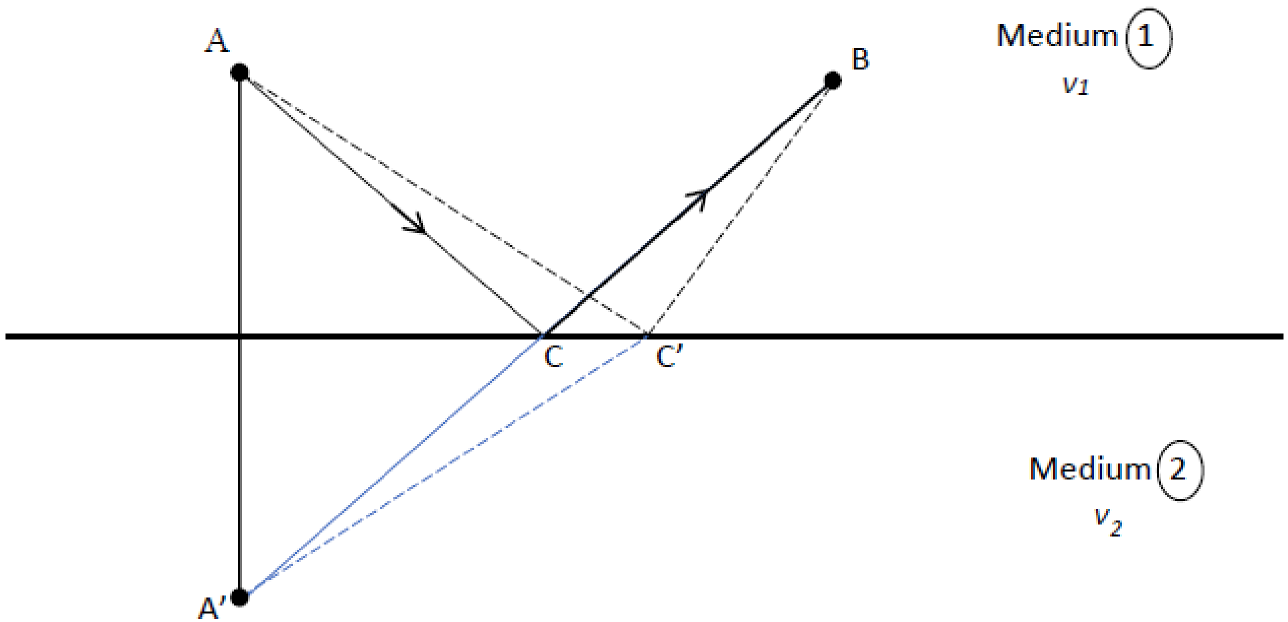

Figure 1: Reflection of the light ray

Least action principle

- Fig.2. depicts the refraction case in which, due to different velocities of light in different media. The shortest path is not the fastest.

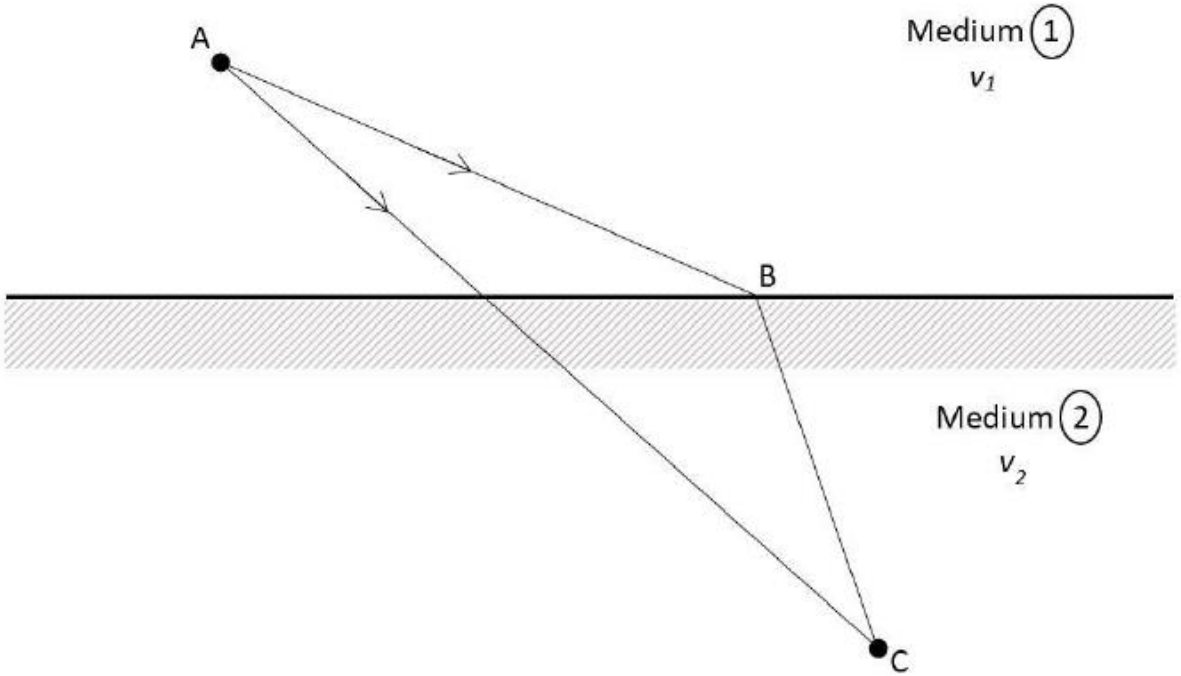

Figure 2: Refraction of the light ray

Least action principle

Law of light reflection

- From the expressions: $dt = \frac{ds}{v} \quad n = \frac{c}{v}$
- where *dt* and *ds* stands from infinitesimal time interval and infinitesimal distance segment, respectively, *v* is light propagation velocity in an arbitrary medium and refraction index *n* is a ratio of light propagation velocity in free space *c* and a material medium, the corresponding functional is

$$t = \frac{1}{c}\int_{s_1}^{s_2} n(x, y)\,ds$$

Least action principle

Law of light reflection

- The problem can be now treated in rectangular coordinates.

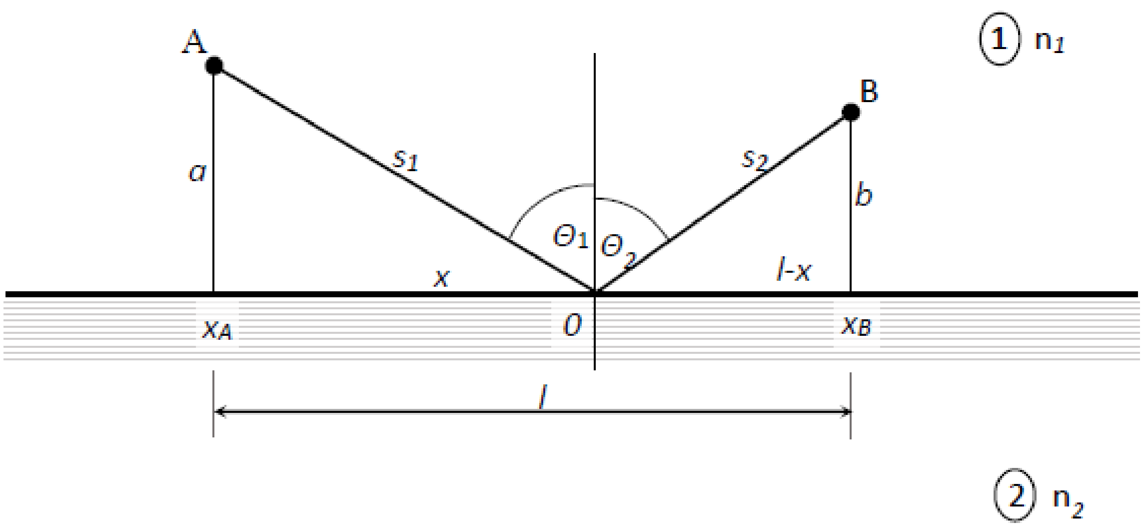

Figure 3: Reflection of the light ray in rectangular coordinates

Least action principle

Law of light reflection

- Note that s_1+s_2 is the total distance travelled by light ray from point *A* to point *B*, while θ_1 and θ_2 stands for incidence and reflection angle, respectively.

- Furthermore, in rectangular coordinates it follows

$$t = \int_{x_A}^{x_B} \frac{1}{c} n(x,y) \sqrt{1+\left(\frac{dy}{dx}\right)^2} dx$$

- Now, the functional (4) is required to be minimal

$$t = \int_{x_A}^{x_B} \frac{1}{c} n(x,y) \sqrt{1+\left(\frac{dy}{dx}\right)^2} dx = \min$$

Least action principle

Law of light reflection

- Note that s_1+s_2 is the total distance travelled by light ray from point *A* to point *B*, while θ_1 and θ_2 stands for incidence and reflection angle, respectively.

- Furthermore, in rectangular coordinates it follows

$$t=\int_{x_A}^{x_B}\frac{1}{c}n\left(x,y\right)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

- Now, the functional is required to be minimal

$$t=\int_{x_A}^{x_B}\frac{1}{c}n\left(x,y\right)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\min$$

Least action principle

Law of light reflection

- Furthermore, one has

$$t=\int_{x_A}^{0}\frac{1}{c}\sqrt{1+\left(\frac{a}{x_A}\right)^2}\,dx+\int_{0}^{x_B}\frac{1}{c}\sqrt{1+\left(\frac{b}{x_B}\right)^2}\,dx$$

and performing straightforward integration yields

$$t=\frac{1}{c}\left(\sqrt{x_A^2+a^2}+\sqrt{x_B^2+b^2}\right)$$

which can be written

$$t=\frac{1}{c}\left(\sqrt{x^2+a^2}+\sqrt{(l-x)^2+b^2}\right)$$

Least action principle

Law of light reflection

- To minimize the time required for light ray to travel from *A* to *B* the following condition is to be satisfied

$$\frac{\delta t}{\delta x} = 0$$

and one obtains

$$\frac{x}{c\sqrt{x^2 + a^2}} - \frac{l - x}{c\sqrt{(l - x)^2 + b^2}} = 0$$

which yields simple condition: $\sin\theta_1 = \sin\theta_2 \Rightarrow \theta_1 = \theta_2$

Least action principle

Law of light refraction

- The problem can be now treated in rectangular coordinates.

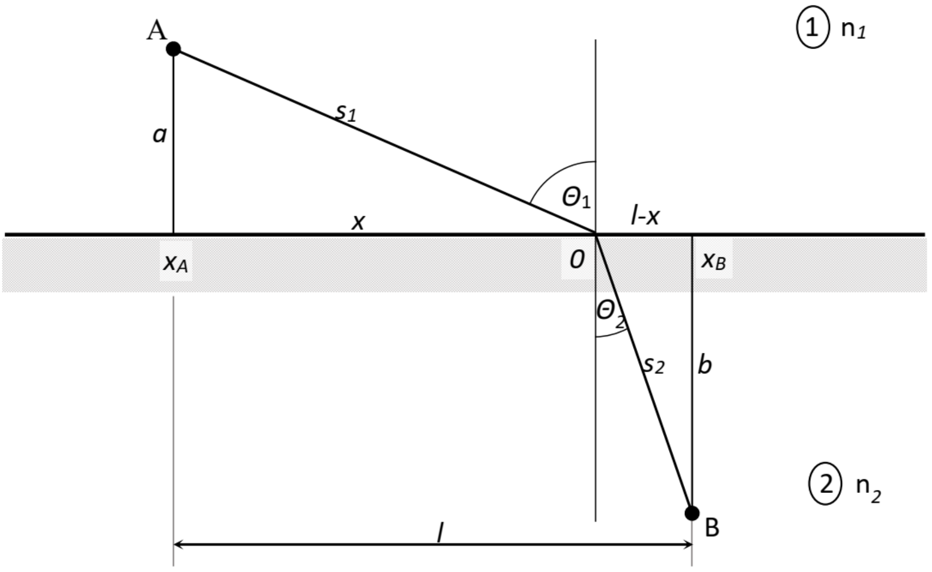

Figure 4: Refraction of the light ray in rectangular coordinates

Least action principle

Law of light refraction

- Again s_1+s_2 is the total distance travelled by light ray from point *A* to point *B*, while θ_1 and θ_2 stands for incidence and refraction angle, respectively.
- Note that n_1 and n_2 denote refraction indices of different media.
- Starting from the request of minimum time it follows

$$t = \int_{x_A}^{0} \frac{n_1}{c} \sqrt{1 + \left(\frac{a}{x_A} \right)^2} dx + \int_{0}^{x_B} \frac{n_2}{c} \sqrt{1 + \left(\frac{b}{x_B} \right)^2} dx$$

Least action principle

Law of light refraction

- Furthermore, one has $t = \dfrac{1}{c}\left(n_1\sqrt{x^2+a^2} + n_2\sqrt{\left(l-x\right)^2+b^2}\right)$

and the request to minimize the time necessary for a light ray traveling from *A* to *B* yields

$$\frac{n_1 x}{c\sqrt{x^2+a^2}} - \frac{n_2\left(l-x\right)}{c\sqrt{\left(l-x\right)^2+b^2}} = 0$$

and one obtains the Snell's laws $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

Time symmetry and conservation of energy

- According to Noether's theorem the consequence of time symmetry is the conservation of energy which can be readily demonstrated by using Lagrangian and Hamiltonian formalism in classical mechanics.

- One-dimensional case pertaining to the study of single particle behaviour is analyzed.

- Consider action $F = \int_{t_1}^{t_2} L dt$

where Lagrangian *L* represents the difference between kinetic and potential energy, respectively, of a particle.

Time symmetry and conservation of energy

- In simple one-dimensional case for a particle of mass *m*, the Lagrangian *L* is given by

$$L = \frac{1}{2} m\dot{x}^2 - W_{pot}(x)$$

where $\dot{x} = \frac{dx}{dt}$

and *x* is the position of particle, while W_{pot} (*x*) denotes the potential energy of the system.

- Lagrangian *L* is explicitly time independent though it is implicitly time dependent as position is a function of time *x*=*x*(*t*).

Time symmetry and conservation of energy

- Now varying action $\delta F = 0$

  one obtains

$$\delta F = \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}} \right) \right] \delta x dt + \frac{\partial L}{\partial \dot{x}} \delta x \Big|_{t_1}^{t_2} = 0$$

- Furthermore, as variation δ*x* vanishes at initial and final time instant, respectively, it follows

$$\frac{\delta F}{\delta x} = \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}} \right) \right] dt = 0$$

  and the Euler-Lagrange equation of motion is $\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}} \right) = 0$

Time symmetry and conservation of energy

- One readily obtains Newton's 2nd law

$$m\frac{d^2x}{dt^2} = -\nabla W_{pot}\left(x\right)$$

where the force on the particle is given by

$$\vec{F} = -\nabla W_{pot}\left(x\right)$$

- Therefore, the particle trajectory minimizing the action (is the one satisfying the Newton equation of motion.

Time symmetry and conservation of energy

- Hamiltonian of the system represents the total energy of the system and can be written as

$$H = \frac{1}{2} m \dot{x}^2 + W_{pot}(x)$$

while the Newton 2nd law can be written as

$$\vec{F} = m \frac{d^2 x}{dt^2} \vec{e}_x$$

- In Hamiltonian notation it can be written:

$$\frac{dp}{dt} = -\frac{\partial H}{\partial x} \qquad \frac{dx}{dt} = +\frac{\partial H}{\partial p} \qquad p = m\dot{x}$$

Time symmetry and conservation of energy

- As one has

$$\frac{\partial L}{\partial \dot{x}}\dot{x} = m\dot{x}$$

the relationship between Hamiltonian and Lagrangian is

$$H = \frac{\partial L}{\partial \dot{x}}\dot{x} - L$$

- Finally, for a system with a time translation symmetry

$$t \rightarrow t' = t + \delta t$$

the Hamiltonian of the system remains invariant.

Time symmetry and conservation of energy

- It is easy to prove that the Hamiltonian is a conserved quantity under a time translation.

- Differentiating with respect to time yields

$$\frac{dH}{dt}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\dot{x}-L\right)=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\dot{x}\right)-\frac{dL}{dt}$$

- Now, one has:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\dot{x}\right)=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)\dot{x}+\frac{\partial L}{\partial \dot{x}}\frac{d\dot{x}}{dt}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)\dot{x}+\frac{\partial L}{\partial \dot{x}}\ddot{x}$$

$$\frac{dL}{dt}=\frac{\partial L}{\partial x}\frac{dx}{dt}+\frac{\partial L}{\partial \dot{x}}\frac{d\dot{x}}{dt}+\frac{\partial L}{\partial t}=\frac{\partial L}{\partial x}\dot{x}+\frac{\partial L}{\partial \dot{x}}\ddot{x}+\frac{\partial L}{\partial t}$$

Time symmetry and conservation of energy

- Furthermore, it can be written

$$\frac{dH}{dt}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)\dot{x}+\frac{\partial L}{\partial \dot{x}}\ddot{x}-\left(\frac{\partial L}{\partial x}\dot{x}+\frac{\partial L}{\partial \dot{x}}\ddot{x}+\frac{\partial L}{\partial t}\right)$$

and one obtains

$$\frac{dH}{dt}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)\dot{x}-\frac{\partial L}{\partial x}\dot{x}-\frac{\partial L}{\partial t}$$

- Furthermore, it follows

$$\frac{dH}{dt}=\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}-\frac{\partial L}{\partial x}\right)\dot{x}-\frac{\partial L}{\partial t}$$

Time symmetry and conservation of energy

- Now, Euler Lagrange equation vanishes

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

and, one obtains

$$\frac{dH}{dt} = -\frac{\partial L}{\partial t}$$

- Time derivative of Lagrangian is zero, as there is no explicit time dependence, therefore the total time derivative of the Hamiltonian is also zero, thus the Hamiltonian is conserved under the time translation.

Time symmetry and conservation of energy

- To sum up; provided the Lagrangian is time invariant the Hamiltonian (total energy of the system) is conserved which represents *the energy conservation law*.
- In other words, energy is conserved if the physical laws do not change with time, i.e. the *action* is invariant under a translation in time.

A note on symmetries

- In a mathematical sense symmetry can be considered as self-similarity of an object, while from the physical point of view symmetry implies invariance or processes that do not change under some circumstances.
- Straightforward example of symmetry (in geometrical sense) is the rotation of some geometrical objects, such as square, or circle.
- Thus, square appears to be the same when rotated for angle of 90°, or it can be stated that square remains unchanged upon such transformations.

A note on symmetries

- Circle, on the other hand, is infinitely symmetrical as it always looks the same, no matter how it is rotated.
- So, being symmetrical is a request that object appears to be the same when subjected to some transformations.
- Gauge symmetries generally represent a change of some parameters of a system which does not influence a system, i.e. does not change the physical laws.

A note on symmetries

- Global symmetries pertain to transformations influencing all points in space-time in a same manner.
- An example of such a symmetry would be a translation by a certain distance along an arbitrary axis of co-ordinate system, or a rotation by an arbitrary angle around the origin of co-ordinate system, as each event in space-time would be influenced in the same way.
- Therefore, provided a system is invariant under such a transformation one deals with a global symmetry.

A note on symmetries

- Therefore, provided a system is invariant under such a transformation one deals with a global symmetry.
- Global symmetries are associated with conservation laws via Noether's theorem, i.e. invariance under time translation yields the conservation of energy, invariance under space translation yields linear momentum conservation, while invariance under rotation results in the angular momentum conservation.
- These invariances imply following symmetries: homogeneity of time and space plus isotropy of space.

A note on symmetries

- The most convenient tool for mathematical representation and analysis of symmetries is the group theory.
- For example, a group *G* representing a collection of elements with multiplication laws has the following properties: closure, associative, identity and inverse.
- Note that local symmetry is associated with electromagnetic theory.

A note on symmetries

- Thus, for source-free Maxwell equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

scalar and vector potential are introduced:

$$\vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A}$$

- The gauge transformations: $\varphi \to \varphi - \frac{\partial \Lambda}{\partial t} \quad \vec{A} \to \vec{A} + \nabla \Lambda$

where Λ is arbitrary scalar function, result in the same electric and magnetic field, respectively.

A note on symmetries

- Electric and magnetic field are gauge invariant.
- By invariant it is assumed that numerical value of a quantity of interest is not changed by a coordinate transformation.
- Conservation of electric charge emerges from this local symmetry (gauge invariance).
- Therefore, physical laws, being gauge invariant, are represented via gauge fields which are not measurable and, consequently, do not have physical meaning.

A note on symmetries

- Nevertheless, gauge fields, being auxilliary functions, are necessary to be defined within classical electromagnetics as, for example Lorentz force can be obtained from the corresponding Lagrangian only if the magnetic vector potential vector is introduced.
- Namely, Lorentz force cannot be derived from Maxwell's equations for non-moving media.

Some philosophical implications

- Noether's theorem associates symmetries with conserved quantities, i.e. symmetries and conservation laws are linked up featuring the use of a minimum principle in nature - the principle of least action.
- The least action principle and the fastest (not the shortest) path of a light ray are often related with Maupertuis attitudes and Leibniz philosophy pertaining to the best of all possible worlds.
- Maupertius, as Leibniz, saw in the least action principle touch of God and sign of the best of all possible worlds.

Some philosophical implications

- It is worth mentioning that Voltaire strongly ridiculed this view in his famous novel "Candid" which was a great success and somehow wiped out Maupertuis and Leibniz views.
- Note that Maupertuis and Leibniz are mentioned in some of Kant's writings.
- Somehow, pure mathematical entities (in this case symmetry properties) influence the laws governing physical world, i.e. objective reality.

Some philosophical implications

- The question is whether **mathematics** represents **ontology**, as a **study of the inner nature of phenomena,** while **physics** deals with **epistemology**, a **study of knowledge one may obtain about the phenomena.**

Some philosophical implications

- These ontic-epistemic relations could be found in;

  - Kant philosophy pertaining to things in themselves (noumena) and their manifestations, or empirical appearences (phenomena) correspondence between platonic world of ideas and non-perfect world of physical objects,

- Leibniz philosophy pertaining to best of all possible worlds,
- or even in Kopenhagen representation of quantum physics.

Some philosophical implications

- According to Noether's 1st theorem every continuous symmetry corresponds to a certain conservation law.
- Therefore, the question is whether Noether's theorem provide a link between ontic and epistemic aspects of a system under consideration.
- In a physical sense one may state that energy represents a property of an object, or system that is conserved under time translation.
- By conserved it is understood that a quantity being considered is not altered throughout some process.

Some philosophical implications

- Thus, space invariance corresponds to the conservation of linear momentum, rotation invariance corresponds to the conservation of angular momentum.
- Gauge symmetry yields the charge conservation.
- Euler Lagrange equations of motion obtained by means of Hamilton principle are unaffected by the addition of a divergence term to the Lagrangian.
- Historically speaking, the developers of the principle of the least action did not have unified attitudes towards the corresponding underlying principle.

Some philosophical implications

- While Fermat claims that Nature follows the path that requires the shortest time, Lagrange refutes any methaphysical meaning of least action principle.

- Anyway, Noether's theorem addresses what remains unchanged, i.e. what quantity is conserved under a time translation.

- Though Noether's theorem shows the correspondence between continuous time translation and the law of energy conservation this correspondence is not only a mathematical description derived in dependence of physical data.

Some philosophical implications

- In the contemporary philosophy of mathematics there are two theoretical approaches that assert a kind of independence of mathematics in relation to the world of physical experience: The Platonist theory and the Theory of innateness of mathematical structures.

- The roots of modern Platoist theory leads to Plato's assertion that the ontological basis of the physical world is geometric figures – imperfect and spatially divisible perfect and time invariant.

Some philosophical implications

- The roots of the theory of innateness of mathematical structures lead to Kant's philosophy stating that mathematics ontologically belongs to the *a priori* domain of research.
- The common point in the ontological aspects of these theories is the assertion of the autonomous independence of mathematics.
- In epistemogical aspects, these two theories are mutually irreconcilably exclusive in a logical sense.

Some philosophical implications

- The Platonist oriented philosophy of mathematics claims that by dealing purely with matehmatics one discovers the inner principles of the structure of the world of physical experience.
- Platonistically interpreted Noether's theorems suggest that the law of energy conservation arising from mathematical symmetries corresponds to ontological and epistemological identity, respectively.
- The theory of innatenes of mathematical structures provides a different connection between mathematical objects and the world of physical experience.

Some philosophical implications

- The correspondence understood in this way, in terms of Kant's philosophy, leads to the philosophy of Thomas Aquinas – *adaequatio intelectus et rei.*
- Dealing with pure matehmatics, a part of the rational structure of the mind is discovered, while applying this mathematical structure in physics allows one to seek the true nature of the universe.
- Such an application of mathematics, on the other hand, is not only a description of a physical state of affairs from the pure mathematics.

Some philosophical implications

- Applying mathematics to physics is one of the rational ways one investigates the truth and there is no ontological and epistemological identity.
- The question is whether mathematics/physics relationship points out to some deeper principles.
- According to Kant's philosophy there are two possibilities: the discovery of some deeper principles of the sensibly accessible world of phenomenality is only a matter of the development of science, or their discovery is noumenally beyond the possibility of the cognitive powers of the mind.

Some philosophical implications

- In Kant's philosophy, the phenomenon is a doubly caused image.
- In a sense of the subject-object relationship; subjectively caused by the apperceptive intuition (*Anschauung*) of space and time, and objectively caused by an unknownable thing in itself (*Ding an Sich*).

Who was Amalie Emmy Noether?

- Amalie Emmy Noether, a great scientist who made crucial contributions in abstract algebra and mathematical physics, was born in Erlangen, Germany on 23 March, 1882.

- Despite numerous obstacles she had experienced as women in academia in those times, she completed her PhD in mathematics, *summa cum laude*, in 1907 at the *University of Göttingen*.

Who was Amalie Emmy Noether?

- Having completed her thesis she collaborated with many prominent mathematicians of that period, such as, David Hilbert, or Hermann Minkovski.
- Women had been allowed into universities in France in 1861, England in 1878, and Italy in 1885, but it was rather high resistance to women presence at universities in Germany at the turn on the century.
- David Hilbert and Felix Klein invited Emmy to give some courses at the *University of Göttingen*, but under Hilbert's name, as at that time women were not allowed to hold official positions.

Who was Amalie Emmy Noether?

- Hilbert and Einstein were deeply frustrated by such a mistreatment of Emmy Noether.

Gentlemen, I do not see that the sex of the candidate is an argument against her admission as a Privatdozent. After all, the faculty senate is not a bathhouse.

David Hilbert

On receiving the new work from Fraulein Noether, I again find it a great injustice that she cannot lecture officially. I would be very much in favour of taking energetic steps in the Ministry to overturn this rule.

Albert Einstein

Who was Amalie Emmy Noether?

- Eventually, University allowed Emmy to give lectures by using her name in 1919 after an oral examination carried out by the faculty members.
- Finally, in 1922 she was appointed as *unofficial associate professor*, which was a purely honorary position.
- In other words, Emmy lectured for free.

Who was Amalie Emmy Noether?

Emmy Noether was one of the most influential mathematicians of the century. The development of abstract algebra, which is one of the most distinctive innovations of twentieth century mathematics, is largely due to her — in published papers, in lectures, and in personal influence on her contemporaries.

Nathan Jacobson

- She gave a proof for her celebrated theorem, mentioned in this paper, in 1915, while it was published 3 years later in 1918.

Who was Amalie Emmy Noether?

- She was a visiting professor in Moscow in 1928-29 and Franfurt in 1930.

- She accepted an offer for a visiting professorship at Bryn Mawr College, Pennsylvania, United States in 1934, and also lectured at the Institute of Advanced Study in Princeton, New Jersey.

When I was called permanently to Gottingen in 1930, I earnestly tried to obtain from the Ministerium a better position for her, because I was ashamed to occupy such a preferred position beside her whom I knew to be my superior as a mathematician in many respects.

Hermann Weyl

Who was Amalie Emmy Noether?

- Amalie Emmy Noether died on 14 April 1935 in Bryn Mawr, Pennsylvania, United States, at the age of 53.
- Emmy's theorem provides a central principle in physics connecting two fundamental concepts in physics; symmetry and conservation, but, finally, the real importance of Emmy's contributions was fully recognised by the end of 20^{th} century.

Who was Amalie Emmy Noether?

- Her achievements in mathematics and physics are of great importance, but she also plays an important role as a women in science in the 1st half of 20th century.
- Namely, together with some other great female scientists such as Mariee Sklodovska-Curie or Lise Meitner, she changed the paradigm of that time that science was a male-dominated activity.
- More than a century later, mathematicians and physicists are still studying and citing her paper from 1918 article and developing her concepts.

Conclusion

- Some physical and philosophical aspects of Noether's theorem have been discussed.
- According to Noether’s theorem every continuous symmetry in action yields conserved quantity.
- The energy is conserved under time translation, linear momentum is conserved if the *action* is invariant under a translation in space.
- If there is rotational invariance of the *action*, the angular momentum is conserved, i.e. the physical laws are the same regardless of the direction.

Conclusion

- Finally, the conservation of electric charge is a consequence of gauge symmetry.
- Therefore, to state that physical laws have a certain symmetry actually means that the *action* is invariant under the transformation associated with that symmetry.
- From the philosophical point of view, a deeper correlation of mathematics and physics has been addressed featuring certain epistemic and ontic aspects, as fundamental laws of nature are represented by the equations with incorporated inherent symmetry.

Conclusion

- In a sense, Noether's theorem relates conservations laws to symmetries of space, time and internal variables thus giving a rise to deeper principles.

Thank you for your kind attention!

Dragan Poljak

University of Split, FESB

dpoljak@fesb.hr

11. SIMPOZIJ

POVIJEST I FILOZOFIJA TEHNIKE

ZAGREB, 14.–15. LIPNJA 2023.

cigre Hrvatska Croatia