

Darko Veljan

More on Zagreb and Mathematics

Abstract: We consider work and life of some of the most world wide known mathematicians and other scientists related to Croatia and in particular to Zagreb. Some of their names and activities are listed in key words below.

Keywords: Mijo Šilobod Bolšić, Ruđer Bošković, Albert Einstein, William (Vilim) Feller, Branko Grünbaum, Sibe Mardešić, Mileva Marić, Vladimir Prelog, Lavoslav Ružička, Nikola Tesla, Vladimir Varićak, Zagreb index, Zagreb school.

Introduction

As a Zagreb born mathematician, I would like to add a few less known ‘gems’ from the mathematical history of Zagreb to the beautiful and relaxing article by B. Dakić and B. Kudelić, A mathematician in Zagreb, *Math. Intelligencer*, online 11 October 2018, [1].



Fig. 1: Vladimir Varićak



Fig. 2: William (Vilko) Feller



Fig. 3: Sibe Mardešić



Fig. 4: Branko Grünbaum

First, let us mention just a couple of Croatian world known mathematicians closely related to Zagreb. Vladimir Varićak (1865 – 1942) [2], William (Vilim) Feller (1906 – 1970) [3], Sibe Mardešić (1927 – 2016) [4], and Branko Grünbaum (1929 – 2018) [5]. Feller was an expert in probability theory, Mardešić in topology and Grünbaum in geometry. They all published many research and popular papers and important textbooks and monographs, and they all started or finished their study of math in Zagreb. In this article we shall concentrate on Varićak. He was among the first (along with H Minkowski, H. Poincaré and E. Borel) who discovered deep relationships between two great theories: Lobachevsky's hyperbolic geometry and Einstein's relativity theory.

Early relativity theory and Vladimir Varićak

After Einstein's famous fundamental papers in 1905 on special theory of relativity, Varićak introduced in 1909/10 the concept of *rapidity* (or *pseudoveLOCITY*) u , related to the ordinary velocity v by $v = c \tanh(u)$, where c is the speed of light. In the meantime, about 1908, Minkowski gave a geometric interpretation of the special theory of relativity as a four-dimensional space-time continuum (manifold) with hyperbolic metric on it. Einstein, and independently Poincaré, proved that the relativistic sum $w = v \oplus v'$ of velocities v and v' acting in the same direction is given by the formula $w = (v + v')/(1 + vv')$, where c is normalized and equal to 1. Varićak's rapidity shows that this formula, in fact, reflects the addition formula for the function \tanh : $\tanh(a + b) = (\tanh a + \tanh b)/(1 + \tanh a \times \tanh b)$. So, for instance, $1/2 \oplus 1/2 = 4/5$, or $1/2 \oplus 2/3 = 7/8$. A simple geometric interpretation is as follows. Consider two points $V(v, 0)$ and $V'(v', 0)$ on the x -axis. Let k be the unit circle with the center at the origin and radius 1, and let $N(0, 1)$ and $S(0, -1)$ be the north and the south pole of k , respectively. Connect S and V by the line and let $A = SV \cap k$, and connect N and V' by the line and let $B = NV' \cap k$ ($A \neq S$, $B \neq N$). Then $W(w; 0)$ is the intersection of the line AB with the x -axis.

Varićak consistently applied hyperbolic trigonometry to prove various claims, for instance, Einstein's formula for Doppler effect, transformation laws of electromagnetic fields in relativity theory and many other related results. His main papers on these topics were published in then leading German journals like *Jahresbericht der Deutschen Mathematiker-Vereinigung* (in 1911), *Physikalische Zeitschrift* (1910) and in *Rad* of Croatian (then Yugoslav) Academy of Science and Art (1909/10). His main book (written in German) *Relativity in 3-dimensional Lobachevsky space*, was published in Zagreb in 1924 and translated into English only in 2007, because its relevance did not diminished even in our days. His works are quoted nowadays as well.

Mileva Marić – Einstein’s spouse and collaborator was educated in Zagreb.

Now comes a little story on Mileva Marić (1875 – 1948) and Zagreb. She was born nearby Novi Sad in Vojvodina, today part of Serbia, but at the time of her birth, it was a part of the Austro-Hungarian Empire (AHE). When she was 16, her father, a clerk in AHE, was transferred from Novi Sad to Zagreb. The family moved to Zagreb in 1891, but Mileva who was good in math and science, could not as a girl simply enrol in a high school in Zagreb where she wanted to continue her studies. So, her father was forced to ask for a special permission for his gifted daughter to enrol the Zagreb Uppertown High School (Gymnasium) for boys. The committee, one of the member of which was Varićak, approved her to enrol in the school, and the next three to four formative years of her education were spent in Zagreb. She got a good chunk of math and physics education in this Uppertown Zagreb (High School) Gymnasium, where her formal math and physics teacher was Ivan Benigar, and her informal math and physics teacher Vladimir Varićak. In 1896 she moved to Zürich and enrolled (after passing the rigorous entrance examinations) in the famous ETH. She then met there Albert Einstein (1879 – 1955), who was at the time 17 years old. They together audited lectures and, in particular, lectures in math by Hermann Minkowski (1869 – 1909) and other prominent professors. In the meantime, Zagreb student Mileva and Munich student Albert started to be not only students, but also good friends, collaborators and eventually got married. Soon, they got their first child (a girl Lieserl, who unfortunately died soon after birth), and later on they got two more children. It is almost certain that they together prepared the revolutionary papers on the special theory of relativity which appeared in 1905 and later.

A detail that shows that Marić and Varićak stayed in friendly relation is that Varićak’s son Svetozar, while student of chemistry at ETH Zürich, 1911 – 1913, lived as a subtenant in the apartment of Mileva and Albert Einstein. Mileva and Albert lived separately from 1914 and formally divorced in 1919. Albert Einstein received the Nobel prize for physics in 1921. After he became world famous,

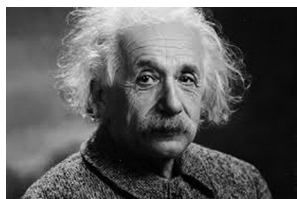


Fig. 5: Albert Einstein

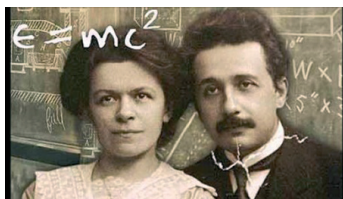


Fig. 6: Mileva Marić and Albert Einstein



Fig. 7: Hermann Minkowski

his professor Minkowski about 1908 said jokingly: “I know Einstein does not know mathematics. Namely, I was his math professor.” What he really meant by that, we’ll never find out. In any case, Mileva Marić has never got any credit for her undoubted mathematical influence on young Albert Einstein. Her good math and physics education in her formative three or four years in Zagreb for sure had some impact on early ideas on relativity theory. In support to this is Albert’s letter to Mileva from 1905 when he exclaimed in a letter to her: “Our paper is accepted for publication!”

In any case, Mileva Marić died without any wide recognition for participating in creating one of the greatest and most influential intellectual achievements of the human kind so far. She was buried in Zürich in her modest grave in 1948. Albert Einstein died in Princeton in 1955 as one of the most famous persons of all time.

Fig. 8: Facsimile of the register of M. Marić. The form contains the following information:

Ime i prezime	Marić, Mileva
Rođeno	18. Dec. 1875
Rođeno mjesto	Titel
Religija	Grčko-istočna
Otac	Miloš, činovnik
Stan	Zagreb, Potoki 72
Učenički broj	68
Učenički razred	III
Učenički predmeti	Matematika, Fizika
Učenički ocjena	Matematika: 8, Fizika: 7
Učenički primjedbe	

Fig. 8: Facsimile of the register of M. Marić

Fig. 9: Facsimile of the register of professors. The form contains the following information:

Ime i prezime	Benigar, Ivan
Rođeno	18. Dec. 1875
Rođeno mjesto	Titel
Religija	Grčko-istočna
Otac	Miloš, činovnik
Stan	Zagreb, Potoki 72
Učenički broj	68
Učenički razred	III
Učenički predmeti	Matematika, Fizika
Učenički ocjena	Matematika: 8, Fizika: 7
Učenički primjedbe	

Fig. 9: Facsimile of the register of professors

Fig. 10: Facsimile of the front page of the register in the acad. year 1891/2. The form contains the following information:

Ime i prezime	Benigar, Ivan
Rođeno	18. Dec. 1875
Rođeno mjesto	Titel
Religija	Grčko-istočna
Otac	Miloš, činovnik
Stan	Zagreb, Potoki 72
Učenički broj	68
Učenički razred	III
Učenički predmeti	Matematika, Fizika
Učenički ocjena	Matematika: 8, Fizika: 7
Učenički primjedbe	

Fig. 10: Facsimile of the front page of the register in the acad. year 1891/2

In Figures 8, 9 and 10 are facsimiles from the Zagreb Archives of the Royal Great Gymnasium in Zagreb of the academic year 1891/92. In Fig. 8 register of Mileva Marić, private student, born in Titel, 18 Dec 1875, homeland: Ugarska (Hungary), religion: Greek-Eastern (grčko-istočna), father: Miloš, clerk of governor office, resides: in Zagreb, Potoki 72. Grades (math and physics): very good and good. Fig 9 Professor of math and physics: Ivan Benigar. Fig 10 shows the front page of the Royal Great Gymnasium in Zagreb for the academic year 1891/92. This Gymnasium was founded in 1607, and in 1669 the Zagreb jesuit academia was established. It is considered as the year of the establishment of the Zagreb University. The Department of Mathematics was established only in 1874.

Just for comparison, in the neighbourhood, the University of Vienna, about 350 km north from Zagreb, was established in 1365. Some, 400 km northeast, the Budapest University was founded in 1635, the Belgrade University (400 km east from Zagreb) in 1838, and Ljubljana University (140 km northwest from Zagreb) in 1919. The first modern Math Gymnasium (High School) in South East Europe was established in Zagreb in 1964. The next one in Belgrade in 1966, then Bucharest in 1967, and Sofia in 1968. The old German Gymnasium (preferably taught math and science) in Budapest started with education in 1908.

Varićak and Bošković

Vladimir Varićak, an informal math teacher of Mileva Marić, was a member of the Croatian (then Yugoslav) Academy of Science and Art. He was also a member of the Serbian Academy of Science and Art and of the Czech Academy of Science. He was a professor of mathematics at the University of Zagreb for more than 40 years. He was also the vice-chancellor of the University of Zagreb 1921 – 1922. He graduated in Zagreb in 1891 and obtained Ph.D. under Czech-born mathematician Karel Zahradnik (1848 – 1916), but was mostly self-educated. First he was a high school teacher of math and physics and then he got a position in 1899 at the University of Zagreb. Since then he taught many classes and lectured on many topics from real and complex analysis, Euclidean and non-Euclidean geometry, Lie theory of integration, set theory, topology, special and general theory of relativity, probability theory and other. Some of his students include William Feller (probability theory), Milutin Milanković (astrophysics), Đuro Kurepa (set theory), Stanko Bilinski (geometry), Željko Marković (differential equations), Ivan Supek (physics), Pavle Papić (topology), Danilo Blanuša (differential geometry and graph theory) to name only a few.

V. Varićak published about 200 papers, books and professional articles in areas of algebraic analysis, non-Euclidean geometry, theory of relativity, in history of science, textbooks and translations and occasional texts. In particular, his research in the history of science about the famous Croatian mathematician, physicist, astronomer and philosopher Rugjer Josip Bošković (1711 – 1787) was very well known and appreciated. Bošković was born in Dubrovnik, studied at Roman jesuit college, and later became professor in Rome, Milan, Brera and Pavia and stayed in Vienna, Milan, Paris and London, where he became a member of the Royal Society. He taught math, astronomy, physics, and particularly optics. In his numerous works, Bošković anticipated non-Euclidean geometry 80 years before Lobachevsky (claiming bravely that the 5. axiom is independent of other geometry axioms). Also, he had some ideas on relativity theory 150 years before Einstein (claiming bravely, though without proof, that

the speed of light is constant regardless of the motion of the source, and that the mass of a body in motion can change; he also anticipated 4-dimensional space of motion). He wrote about the model of atoms and 160 years before Bohr invented the model. Bošković made various contributions in spherical geometry, astronomy (Gauss appreciated his works in astronomy), mechanics, statics and optics etc. As an elementary fact, he rediscovered Pappus's definition of conics as a set of points such that the ratio of distances to the given point (focus) and the given line (directrix) is constant. He contemplated about infinitely big and small quantities anticipating what some 120 years later became known as Dedekind cuts, essential in axiomatization of real numbers. Also, long before Cauchy, at the time of Euler, Bošković also clarified some concepts of the Newton-Leibniz ('infinitesimal') calculus. The method of least squares was invented by Bošković and reinvented later by Gauss. Werner Heisenberg (1901 – 1979) the Nobel prize laureate for physics in 1932, called Bošković 'Croatian Leibniz'. Bošković's main book *Theory of Natural Philosophy* was published in Latin in Vienna in 1758 and the second edition was published in 1763 in Venice. The first English edition appeared in London in 1922. Bošković has many memorial tablets and statues all over Croatia, but also abroad: in Rome (Accademia Nazionale dei Lincei), in Vienna since 1952 (see [6]), in Paris since 2013 and in Milan since 2017.



Fig. 11: Rugjer Josip Bošković

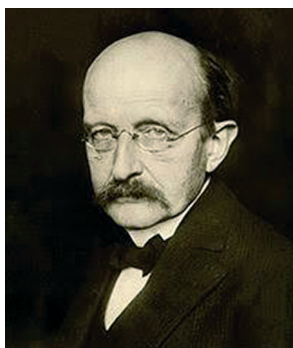


Fig. 12: Max Planck

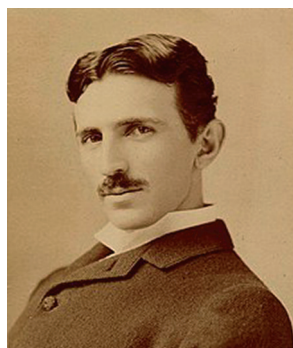


Fig. 13: Nikola Tesla

Varićak thoroughly investigated Bošković's work, visited many archives and wrote more than 20 scientific papers on Bošković and his work. Varićak died in Zagreb in March 1942. On 15 September 1942, the legendary German physicist and Nobelist Max Planck (1858 – 1947) visited Zagreb and delivered a lecture on Technical University entitled *The importance and limits of exact natural sciences*, with the emphasis on relativity theory and quantum physics.

Nikola Tesla

In January 1943, another great Croatian-American inventor and scientist Nikola Tesla (1856 – 1943) died in New York City. Tesla was born in a village Smiljan about 140 km from Zagreb. He was also fascinated with achievements of his countryman Rugjer Bošković. In the very center of Zagreb there are statues of Bošković and Tesla and streets named after both Bošković and Tesla, and a little street in the new part of Zagreb is named after Vladimir Varićak. The *Institute RugjerBošković* in Zagreb is well known research institute, and Zagreb factory *Ericsson-Tesla* produces various electronic devices. Nikola Tesla on his tour from USA to Europe visited Zagreb and in the city hall delivered a lecture on 24 May 1892 about his inventions and proposed that Zagreb would be the first city in the world with public lighting based on his newly invented alternate current. Due to the lack of funds, the proposal was denied. Instead, the coast city of Šibenik in Dalmatia (region of Croatia) soon introduced the first alternate electric plant in this part of Europe. Interestingly, in the beginning, Tesla was not very much in favour of the relativity theory, because it did not fit into his theory of energy, but later on he revised his views.

Zagreb index, Zagreb school and Zagreb textbooks

We end this little story on Zagreb and math with a couple of remarks. The first is on *Zagreb index*. This is a well established concept in chemical graph theory, defined (about 1972) as the sum of squared degrees of vertices of the (molecule) graph. Since then more than 300 papers deal with this concept. This shows an active cooperation between mathematicians and chemists in Croatia (and countries around). Perhaps this is due to the fact that Croatia has two Nobel prize laureates in chemistry: Lavoslav Ružička (1887 – 1976), Vukovar born and later professor at ETH in Zürich, Nobel prize winner in 1939, and Vladimir Prelog (1906 – 1998), Sarajevo born and later chemistry professor in Zagreb and then at ETH in Zürich, Nobel prize winner in 1976. Many streets in Croatia and particularly in Zagreb are named after them. A high school in Zagreb with education mostly oriented to natural sciences is named Gymnasium *Vladimir Prelog*. Let us only mention here that Drago Grdenić (1919 – 2018) a chemistry professor at Zagreb University was the oldest academician (almost 100 years old) at the Croatian Academy of Science and Art and lived just 50 meters from the Tesla statue in Zagreb. Even when he was alive, the Academy erected his statue in his birthplace of Bjelovar, about 70 km from Zagreb.

In years 1961 – 2005, there were very active Seminars held in a few disciplines at the Department of Mathematics of the Zagreb University and each had a nickname “Zagreb school”. It was in topology, functional analysis, representation theory, geometry and later also in combinatorics, chemical graph theory, probability theory and number theory. They, as well as some other Seminars are still active but the initial activities as always were more intensive.

Finally, we should say a few words about the oldest math textbook in Croatian language. The title is *Arithmetika Horvatszka* (Croatian Arithmetic). The author was Mijo Šilobod Bolšić, a priest, teacher and an educational and cultural activist. The book appeared in Zagreb back in 1758, the same year when Bošković’s capital work appeared in Vienna. Šilobod’s work was the first math book in South-East Europe written not in Latin, German or Hungarian, and was not a translation of existing textbooks or old Greek or Arabian texts, but on the people’s language – Croatian, understandable to ordinary folks: pupils, students, workers, peasants, merchants, soldiers, housewives etc. In fact, the book was written in an old Zagreb kajkavian dialect. The book was not a formal textbook, but was written in the form of a dialogue between teacher and reader asking questions. A typical example, if a book and a pen cost together 110 (units), and the book costs 100 more than the pen, tell me as quick as you can, what is the cost of the pen. In addition, there are some logic puzzles. For instance, two fathers and two sons caught three rabbits, and each got one rabbit. Tell me, how is this possible? (Answer: grandfather, son and grandson). As a cultural heritage, the book was reprinted in 2008. A street in Zagreb is also named after Mijo Šilobod Bolšić.

From 1758 to our days many math textbooks, monographs, conference proceedings and research journals, papers or math books and popular papers appeared in Croatian language all over Croatia, in cities like Split, Rijeka, or Osijek, but mostly in its capital – Zagreb.

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Više o Zagrebu i matematici

Darko Veljan

Sažetak: Razmatra se rad i život nekolicine svjetski poznatih matematičara i drugih znanstvenika povezanih s Hrvatskom, posebice Zagrebom. Neka imena popisana su u ključnim riječima.

Ključne riječi: Mijo Šilobod Bolšić, Ruđer Bošković, Albert Einstein, William (Vilim) Feller, Branko Grünbaum, Sibe Mardešić, Mileva Marić, Vladimir Prelog, Lavoslav Ružička, Nikola Tesla, Vladimir Varićak, Zagreb index, Zagreb school.

